SIDDHARTH GROUP OF INSTITUTIONS :: PUTTUR
Siddharth Nagar, Narayanavanam Road - 517583
QUESTION BANK (DESCRIPTIVE)
Subject with Code: PROBABILITY\&STATISTICS (16HS613) Branches: CIVIL,EEE,MECH,CSE,CS\&IT,AE
Year \&Sem: II-B.Tech\& II-Sem
Regulation: R16

## UNIT -I

1. a) Two cards are selected at random from 10 cards numbered 1 to 10 . Find the probability that the sum is even if (i) The two cards are drawn together. (ii) The two cards drawn one after other with replacement.
[6 M]
b) Determine (i) $P(B / A)$ (ii) $P\left(A / B^{C}\right)$ if A and B are events with $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$,

$$
\begin{equation*}
P(A \cup B)=\frac{1}{2} \tag{4M}
\end{equation*}
$$

2. a) State and prove Baye's theorem.
b) The probability that students $A, B, C, D$ solve the problem are $\frac{1}{3}, \frac{2}{5}, \frac{1}{5}$ and $\frac{1}{4}$ respectively If all of them try to solve the problem, what is the probability that the problem is solved. [4M]
3. Two dice are thrown. Let $X$ assign to each point $(a, b)$ in $S$ the maximum of its numbers i.e, $X(a, b)=\max (a, b)$. Find the probability distribution. $X$ is a random variable with $X(s)=\{1,2,3,4,5,6\}$. Also find the mean and variance of the distribution.
4. A random variable $X$ has the following probability function

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | 0 | K | 2 K | 2 K | 3 K | $\mathrm{~K}^{2}$ | $2 \mathrm{~K}^{2}$ | $7 \mathrm{~K}^{2}+\mathrm{K}$ |

Determine (i) K (ii) Evaluate $\mathrm{P}(\mathrm{X} \geq 6)$ and $\mathrm{P}(0<\mathrm{X}<5)$ (iii) if $\mathrm{P}(\mathrm{X} \leq \mathrm{K})>1 / 2$, find the minimum value of $K$ (iv) variance.
5. Probability density function of a random variable X is $f(x)=\left\{\begin{array}{l}\frac{1}{2} \sin x, \text { for } 0 \leq x \leq \pi \\ 0, \text { elsewhere }\end{array}\right.$. Find the mean, mode and median of the distribution and also find the probability between 0 and $\pi / 2 \cdot[10 \mathrm{M}]$
6. a)A businessman goes to hotels $\mathrm{X}, \mathrm{Y}, \mathrm{Z}, 20 \%, 50 \%, 30 \%$ of the time respectively. It is known that $5 \%$, $4 \%, 8 \%$ of the rooms in $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ hotels have faulty plumbings. What is the probability that businessman's room having faulty plumbing is assigned to hotel Z ?
b) If A and B are independent events then i) $\bar{A}$ and $\bar{B}$ are also independent events .
ii) $\bar{A}$ and B are also independent events.
[5 M]
7. a)In a certain town $40 \%$ have brown hair, $25 \%$ have brown eyes and $15 \%$ have both brown hair and brown eyes. A person is selected at random from the town.
i) If he has brown hair, what is the probability that he has brown eyes also?
ii )If he has brown eyes, determine the probability that he does not have brown hair? [5 M]
b) If X is a continuous random variable and k is a constant, then prove that
i) $\operatorname{Var}(X+k)=\operatorname{Var}(X)$
ii) $\operatorname{Var}(k X)=k^{2} \operatorname{Var}(X)$
8. Two dice are thrown $X$ assign to each point if $S$ is the sum of the variable on the faces. Find mean and variance of the random variable.
9. a) In a certain college $25 \%$ of boys and $10 \%$ of girls are studying mathematics. The girls
constitute $60 \%$ of the student body. (a) What is the probability that mathematics is being studied? (b) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl?
b) Find the mean and variance of the uniform probability distribution given by $f(x)=\frac{1}{n}$ for $x=1,2, \ldots, n$.
10. The probability density $f(x)$ of a continuous random variable is given by $f(x)=c e^{-|x|},-\infty<x<\infty$.

Show that $c=1 / 2$ and find that the mean and variance of the distribution. Also find the probability that the variate lies between 0 and 4 .

## UNIT-II

1. a) Derive mean and variance of Binomial distribution.
b) $20 \%$ of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at random (i) one is defective
(ii) $p(1<x<4)$
[4 M]
2. a) Fit a Binomial distribution to the following frequency distribution:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f$ | 13 | 25 | 52 | 58 | 32 | 16 | 4 |

b) The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$. Find $p(X \geq 1)$.
3. a) Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys. Assume equal probabilities for boys and girls.
b) Two dice are thrown five times. Find the probability of getting 7 as sum i) at least once
(ii) $p(1<x<5)$
4. a) Derive mean and variance of poisson distribution.
b) If $2 \%$ of light bulbs are defective. Find the probability that (i) At least one is defective
(ii) $p(1<x<8)$ in a sample of 100
5. a) Fit a Poisson distribution to the following frequency distribution:

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f$ | 109 | 65 | 22 | 3 | 1 |

b) If the mean of a Poisson distribution is 1.8 then find $p(X>1)$.
6. a) An insurance agent policies of 5 men all of identical age and good in health. The probability that a man of this age will be alive 30 years is $2 / 3$. Find the probability that in 30 years.
(i) At least one man (ii) Almost three will be alive
b) If X is a Poisson variate such that $P(X=1) \frac{3}{2}=P(X=3)$, find (i) $P(X \geq 1)$ (ii) $P(X \leq 3)$
7. Derive mean and variance of Normal distribution.
8. Find the mean and variance of a Normal distribution in which $7 \%$ of items are under 35 and $89 \%$ are under 63.
9. Find the mean and variance of a Normal distribution in which $31 \%$ of items are under 45 and $8 \%$ are over 63 .
[10 M]
10. In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5 .Assuming the distribution to be normal find (i) How many students score between 12 and 15. (ii) How many students score above 18 ? (iii) How many students score below 18 ?
[10 M]

## UNIT-III

1. a) A Sample of 64 students has a mean weight of $70 \mathrm{k} . g m \mathrm{~s}$. Can this be regarded as a sample from a population with mean weight $65 \mathrm{k} . g m \mathrm{~s}$ and S.D $25 \mathrm{k} . g m s$ with level of significance. [5M] b) In two large populations, there are $30 \%$, and $25 \%$ respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two population. [5M]
2. a) The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches. [ 5 M ]
b) Experience had shown that $20 \%$ of a manufactured product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level.
[5M]
3. a) A sample of 900 members has a mean of 3.4 cms and S.D 2.61 cms . Is the sample from a large population of mean 3.25 cm and S.D 2.61 cms . If the population is normal and its mean is unknown find the $95 \%$ fiducial limits of true mean.
[5M]
b)In a big city 325 men out of 600 menwere found to be smokers.Does this information support the conclusion that majority of men in this city are smokers?
[5M]
4. On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups, the upper $30 \%$ and the remaining $70 \%$. Consider the first question of the examination. Among the first group, 40 had the correct answer, whereas among the second group, 80 had the correct answer. On the basis of these results, can one conclude that the first question is not good at discriminating ability of the type being examined here?
[10M]
5. The nicotine in milligrams of two samples of tobacco were found to be as follows.

| Sample A | 24 | 27 | 26 | 23 | 25 | --- |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample B | 29 | 30 | 30 | 31 | 24 | 36 |

Can it be said that the two samples have come from the same normal population.
[10M]
6. A random sample of 10 boys had the following I.Q's : 70,120, $110,101,88,83,95,98,107$ and 100
a)Do these data support the assumption of a population mean I.Q of 100 ?
b) Find a reasonable range in which most of the mean I.Q values of samples of 10 boys lie.
7. a) Blood pressure of 5 women before and after intake of a certain drug are given below

| Before | 110 | 120 | 125 | 132 | 125 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| After | 120 | 118 | 125 | 136 | 121 |

Test whether the significant change in blood pressure at $1 \%$ level of significance.
b) A sample of 26 bulbs gives a mean life of 990 hours with a S.D of 20 hours. The manufacturer claims that the mean life of bulbs is 1000 hours. Is the sample not up to the standard.
8. A die is thrown 264 times with the following results. Show that the die is biased. $\left(\psi^{2}=11.07\right.$ at 5 d.f \& $5 \%$ L.S $)$

| Number <br> on the die | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 40 | 32 | 28 | 58 | 54 | 52 |

9. a)The following table gives the classification of 100 workers according to sex and nature of work. Test whether the nature of work is independent of the worker $\left(\psi^{2}=3.84\right.$ at 1d.f $)$

|  | Stable | Unstable | Total |
| :---: | :---: | :---: | :---: |
| Males | 40 | 20 | 60 |
| Females | 10 | 30 | 40 |
| Total | 50 | 50 | 100 |

b) The measurements of the output of two units have given the following results. Assuming that both samples have been obtained from the normal populations at $10 \%$ significant level, test whether the two populations have the same variance.
[10M]

| Unit A | 14.1 | 10.1 | 14.7 | 13.7 | 14.0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Unit B | 14.0 | 14.5 | 13.7 | 12.7 | 14.1 |

10. Two random samples reveal the following results:
[10M]

| Sample | Size | Sample Mean | Sum of squares of deviations from the mean |
| :---: | :---: | :---: | :---: |
| 1 | 10 | 15 | 90 |
| 2 | 12 | 14 | 108 |

Test whether the samples came from the same normal population.

## UNIT-IV

1. Define ANOVA.Describe briefly the technique of ANOVA for one-way classification. [10M]
2. Describe briefly the technique of ANOVA for Two-way classification.
[10M]
3. Set up an analysis of variance table for the following per acre production data for three varieties of wheat, each grown on 4 plots and state if the variety differences are significant.
[10M]

| Plot of <br> land | Per acre production data |  |  |
| :---: | :---: | :---: | :---: |
|  | A | Bariety of Wheat |  |
|  | 6 | C |  |
| 2 | 7 | 5 | 5 |
| 3 | 3 | 3 | 3 |
| 4 | 8 | 7 | 4 |

4. Three samples each of size 5 were drawn from 3 uncorrelated normal populations with equal variances. Test the hypothesis that the population means are equal at $5 \%$ level.
[10M]

| Sample-1 | 10 | 12 | 9 | 16 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Sample-2 | 9 | 7 | 12 | 11 | 11 |
| Sample-3 | 14 | 11 | 15 | 14 | 16 |

5. An agriculturist wants to test the effects of four different fertilizers A, B, C, D on the yield of paddy. In order to eliminate sources of error due to variabilities in self-fertility, he used the fertilizers in Latin square arrangements given below where the numbers indicate yields in quintals per unit area. Perform an analysis of variance to decide whether there is a difference between the fertilizers at $5 \%$ level of significance.

| A18 | C21 | D25 | B11 |
| :--- | :--- | :--- | :--- |
| D22 | B12 | A15 | C19 |
| B15 | A20 | C23 | D24 |
| C22 | D21 | B10 | A27 |

6. Analyse the variance in Latin square of yields (in quintels) of wheat where $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ represent the different manners used
[10M]

| D222 | A221 | C223 | B222 |
| :--- | :--- | :--- | :--- |
| B224 | C223 | A222 | D225 |
| A220 | B219 | D220 | C221 |
| C222 | D223 | B221 | A222 |

Test whether the different manures used have given significantly different yields.
7. A former applies three types of fertilizers on 4 separate plots. The figure on yields per acre are tabulated below
[10M]

| Plots | YIELD |  |  |  | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Fertilizers | A | B | C | D | T |
| Nitrogen | 6 | 4 | 8 | 6 | 24 |
| Potash | 7 | 6 | 6 | 9 | 28 |


| Phosphates | 8 | 5 | 10 | 9 | 32 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Total | 21 | 15 | 24 | 24 | 84 |
| Grand total |  |  |  |  |  |

Find out if the plots are materially different in fertility, as also, if three fertilizers make any material difference in yields.
8. Set up an analysis of variation table for the following two-way design results [10M] Per Acre Production Data of Wheat

|  | (in metric tonnes) |  |  |
| :---: | :---: | :---: | :---: |
| Varieties of seeds | A | B | C |
| Varieties of Fertilizers | 6 | 5 | 5 |
| W | 7 | 5 | 4 |
| X | 3 | 3 | 3 |
| Y | 8 | 7 | 4 |

Also state whether variety differences are significant at 5 per cent level.
9. a) Define R.B.D and L.S.D .
b) Describe briefly the technique of ANOVA for Two-way classification.
10. Three different machines are used for a production. On the basis of the outputs, test whether the Machines are equally effective.
[10M]

| OUTPUTS |  |  |
| :--- | :--- | :--- |
| Machine 1 | Machine 2 | Machine 3 |
| 10 | 9 | 20 |
| 5 | 7 | 16 |
| 11 | 5 | 10 |
| 10 | 6 | 4 |

## UNIT-V

1. The following data shows the values of sample mean and range for 10 samples for size 6 each. Calculate the central limits for mean chart and R- chart and draw the control charts and comment on the state of control. [10M]

| Sample no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $(\bar{x})$ | 43 | 49 | 37 | 44 | 45 | 37 | 51 | 46 | 43 | 47 |
| Range (R) | 5 | 6 | 5 | 7 | 7 | 4 | 8 | 6 | 4 | 6 |

2. The following are the samples means and ranges for 10 samples each of size 5 . Construct a mean and range chart and determine whether the process is in control.
[10M]

| Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $\bar{X}$ | 20 | 34 | 45 | 39 | 26 | 29 | 13 | 34 | 37 | 23 |
| Range | 23 | 39 | 15 | 05 | 20 | 17 | 21 | 11 | 40 | 10 |

3. Ten samples each of size 5 drawn at regular intervals from a manufacturing process. The sample means chart and their range are given below.
[10M]

| Sample no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean $(\bar{x})$ | 49 | 45 | 48 | 53 | 39 | 47 | 46 | 39 | 51 | 45 |
| Range (R) | 7 | 5 | 7 | 9 | 5 | 8 | 8 | 6 | 7 | 6 |

Calculate the control limits in respect of mean chart and R - chart comment on the state of control.
4. The following are the figures give the number of defectives in 20 samples, containing 2000 items.
$425,430,216,341,225,322,280,306,337,305,356,402,216,264,126,409,193,326,280,389$
Draw control chart for fraction defective and comment on the state of control of the Process.
5. Samples of 100 tubes are drawn randomly from the output of a process that produces several thousand units daily. Sample items are inspected for quality and defective tubes are rejected. The result of 15 samples are shown below:

| Sample No. | No. of Defective tubes | Sample No. | No. of Defective tubes |
| :--- | :--- | :--- | :--- |
| 1 | 8 | 9 | 10 |
| 2 | 10 | 10 | 13 |
| 3 | 13 | 11 | 18 |
| 4 | 9 | 12 | 15 |
| 5 | 8 | 13 | 12 |
| 6 | 10 | 14 | 14 |
| 7 | 14 | 15 | 9 |
| 8 | 6 |  |  |

On the basis of information given above prepare a control chart for fraction defective. What conclusion do you draw from the control chart?
6. The following are the figures of defectives in 22 lots each containing 2000 rubber belts:
[10M] 425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356 402, 216, 264, 126, 409, 193, 326, 280, 389, 451, 420
Draw control chart for fraction defective and comment on the state of control of the Process.
7. Assume that 20 half liter milk bottles are selected at random from a process. The number of air bubble observed from the bottles is given in the table
[10M]

| Bottle <br> number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Defects | 4 | 5 | 7 | 3 | 3 | 5 | 6 | 2 | 4 | 8 | 3 | 5 | 4 | 3 | 4 | 5 | 3 | 7 | 6 | 13 |

Draw the control chart for the above data.
8. The number of weekly customer complaints are monitored at a large hotel using a C-chart. Complaints have been recorded over the past twenty weeks. Develop three sigma contol limits using the following
data

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No.of <br> complaints | 3 | 2 | 3 | 1 | 3 | 3 | 2 | 1 | 3 | 1 | 3 | 4 | 2 | 1 | 1 | 1 | 3 | 2 | 2 | 3 |

9. A textile company wishes to implement a quality control program on a certain garment with respect to the number of defects found in the final production. A garment was sampled on 33 consecutive hours of production. The number of defects found per garment is given hereunder.
[10M]
Defects: 5,1,7,1,0,2,3,4,0,3,2,4,3,4,4,1,4,2,1,3,4,3,11,3,7,8,5,6,1,2,4,7,3
Compute the upper and lower 3 -sigma control limits for monitoring the number of defects.
10. a) Write the constructions of mean, range, p and c -charts.
b) write the causes of variations.
